

NAG Fortran Library Routine Document

F08QXF (CTREVC/ZTREVC)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F08QXF (CTREVC/ZTREVC) computes selected left and/or right eigenvectors of a complex upper triangular matrix.

2 Specification

```

SUBROUTINE F08QXF (JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,
1              MM, M, WORK, RWORK, INFO)
ENTRY          ctrevc (JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,
1              MM, M, WORK, RWORK, INFO)
INTEGER        N, LDT, LDVL, LDVR, MM, M, INFO
real          RWORK(*)
complex      T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(*)
LOGICAL        SELECT(*)
CHARACTER*1    JOB, HOWMNY

```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine computes left and/or right eigenvectors of a complex upper triangular matrix T . Such a matrix arises from the Schur factorization of a complex general matrix, as computed by F08PSF (CHSEQR/ZHSEQR), for example.

The right eigenvector x , and the left eigenvector y , corresponding to an eigenvalue λ , are defined by:

$$Tx = \lambda x \text{ and } y^H T = \lambda y^H \text{ (or } T^H y = \bar{\lambda} y).$$

The routine can compute the eigenvectors corresponding to selected eigenvalues, or it can compute all the eigenvectors. In the latter case the eigenvectors may optionally be pre-multiplied by an input matrix Q . Normally Q is a unitary matrix from the Schur factorization of a matrix A as $A = QTQ^H$; if x is a (left or right) eigenvector of T , then Qx is an eigenvector of A .

The eigenvectors are computed by forward or backward substitution. They are scaled so that $\max(|\operatorname{Re}(x_i)| + |\operatorname{Im}(x_i)|) = 1$.

4 References

Golub G H and van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: JOB – CHARACTER*1 *Input*

On entry: indicates whether left and/or right eigenvectors are to be computed as follows:

if JOB = 'R', only right eigenvectors are computed;

if JOB = 'L', only left eigenvectors are computed;

if JOB = 'B', both left and right eigenvectors are computed.

Constraint: JOB = 'R', 'L' or 'B'.

- 2: HOWMNY – CHARACTER*1 *Input*
On entry: indicates how many eigenvectors are to be computed as follows:
 if HOWMNY = 'A', all eigenvectors (as specified by JOB) are computed;
 if HOWMNY = 'B' or 'O', all eigenvectors (as specified by JOB) are computed and then pre-multiplied by the matrix Q (which is overwritten);
 if HOWMNY = 'S', selected eigenvectors (as specified by JOB and SELECT) are computed.
Constraint: HOWMNY = 'A', 'B', 'O' or 'S'.
- 3: SELECT(*) – LOGICAL array *Input*
Note: the dimension of the array SELECT must be at least $\max(1, N)$ if HOWMNY = 'S' and at least 1 otherwise.
On entry: SELECT specifies which eigenvectors are to be computed if HOWMNY = 'S'. To obtain the eigenvector corresponding to the eigenvalue λ_j , SELECT(j) must be set .TRUE..
 SELECT is not referenced if HOWMNY = 'A', 'O' or 'B'.
- 4: N – INTEGER *Input*
On entry: n , the order of the matrix T .
Constraint: $N \geq 0$.
- 5: T(LDT,*) – **complex** array *Input/Output*
Note: the second dimension of the array T must be at least $\max(1, N)$.
On entry: the n by n upper triangular matrix T , as returned by F08PSF (CHSEQR/ZHSEQR).
On exit: T is used as internal workspace prior to being restored and hence is unchanged.
- 6: LDT – INTEGER *Input*
On entry: the first dimension of the array T as declared in the (sub)program from which F08QXF (CTREVC/ZTREVC) is called.
Constraint: $LDT \geq \max(1, N)$.
- 7: VL(LDVL,*) – **complex** array *Input/Output*
Note: the second dimension of the array VL must be at least $\max(1, MM)$ if JOB = 'L' or 'B' and at least 1 if JOB = 'R' .
On entry: if HOWMNY = 'O' or 'B' and JOB = 'L' or 'B', VL must contain an n by n matrix Q (usually the matrix of Schur vectors returned by F08PSF (CHSEQR/ZHSEQR)). If HOWMNY = 'A' or 'S', VL need not be set.
On exit: if JOB = 'L' or 'B', VL contains the computed left eigenvectors (as specified by HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues.
 VL is not referenced if JOB = 'R'.
- 8: LDVL – INTEGER *Input*
On entry: the first dimension of the array VL as declared in the (sub)program from which F08QXF (CTREVC/ZTREVC) is called.
Constraints:
 $LDVL \geq \max(1, N)$ if JOB = 'L' or 'B',
 $LDVL \geq 1$ if JOB = 'R'.

- 9: VR(LDVR,*) – **complex** array Input/Output
Note: the second dimension of the array VR must be at least $\max(1, MM)$ if JOB = 'R' or 'B' and at least 1 if JOB = 'L' .
On entry: if HOWMNY = 'O' or 'B' and JOB = 'R' or 'B', VR must contain an n by n matrix Q (usually the matrix of Schur vectors returned by F08PSF (CHSEQR/ZHSEQR)). If HOWMNY = 'A' or 'S', VR need not be set.
On exit: if JOB = 'R' or 'B', VR contains the computed right eigenvectors (as specified by HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues.
 VR is not referenced if JOB = 'L'.
- 10: LDVR – INTEGER Input
On entry: the first dimension of the array VR as declared in the (sub)program from which F08QXF (CTREVC/ZTREVC) is called.
Constraints:
 $LDVR \geq \max(1, N)$ if JOB = 'R' or 'B',
 $LDVR \geq 1$ if JOB = 'L'.
- 11: MM – INTEGER Input
On entry: the number of columns in the arrays VL and/or VR. The precise number of columns required, m , is n if HOWMNY = 'A', 'O' or 'B'; if HOWMNY = 'S', m is the number of selected eigenvectors (see SELECT), in which case $0 \leq m \leq n$.
Constraint: $MM \geq m$.
- 12: M – INTEGER Output
On exit: m , the number of selected eigenvectors. If HOWMNY = 'A', 'O' or 'B', M is set to n .
- 13: WORK(*) – **complex** array Workspace
Note: the dimension of the array WORK must be at least $\max(1, 2 * N)$.
- 14: RWORK(*) – **real** array Workspace
Note: the dimension of the array RWORK must be at least $\max(1, N)$.
- 15: INFO – INTEGER Output
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

If x_i is an exact right eigenvector, and \tilde{x}_i is the corresponding computed eigenvector, then the angle $\theta(\tilde{x}_i, x_i)$ between them is bounded as follows:

$$\theta(\tilde{x}_i, x_i) \leq \frac{c(n)\epsilon\|T\|_2}{sep_i}$$

where sep_i is the reciprocal condition number of x_i .

The condition number sep_i may be computed by calling F08QYF (CTRSNA/ZTRSNA).

8 Further Comments

The real analogue of this routine is F08QKF (STREVC/DTREVC).

9 Example

See Section 9 of the document for F08NVF (CGEBAL/ZGEBAL).
